1. Given that $z_1 = 1 + 3i$ and $z_2 = -3 + 2i$, find

(i) $|z_1|$, [1]

(ii) $\arg z_2$ [1]

(iii) $z_1 z_2$ [2]

(iv) $\frac{z_1}{z_2}$ [2]

Show the complex numbers z_1 and z_2 on the same Argand diagram, clearly labelling $|z_1|$ and $\arg z_2$. [2]

(nov2003)

2. (a) Express $Z = \frac{2+i}{3-i}$ in modulus argument form. Hence find in their simplest form the moduli and arguments of the numbers,

(i) Z^2,

(ii) $\frac{1}{Z}$. [6]

(b) (i) Shade the area represented on an Argand diagram by

$|Z - 1 + 2i| < 3$. [2]

(ii) Sketch the locus of Z if

$\arg(Z - 1) - \arg(Z + 1) = \frac{\pi}{6}$ [3]

(june2004)

3. Given that $Z = 4 - 2i$, find

(i) $|Z|$ and $\arg Z$, [2]

(ii) $\frac{Z}{\bar{Z}}$ in the form $a + bi$, where $\bar{Z}$ represents the conjugate of Z and a and b are real numbers. [2]

(nov2004)

4. The complex numbers Z_1 and Z_2 are given by $Z_1 = 1 + ai$ and $Z_2 = -b - i$, respectively, where a and b are real and positive. Given that $Z_1 Z_2 = 5 - 4i$, find

(i) $|Z_1 Z_2|$, [1]

(ii) $\arg(Z_1 Z_2)$ [1]

(iii) the exact values of a and b. [6]

3 The complex number $z = 2 + 3i$ has modulus k and argument $\propto$.

(a) Determine the value of k and the value of $\propto$. [2]

(b) ω is the complex number $z + 3iz$. Find ω in the form $a + ib$ and hence represent ω on the Argand diagram. [3]

14 **(a)** The complex numbers a and b are given by $a = 4 - 7i$ and $b = 1 + i$.

Find

(i) $\left|\frac{a}{b}\right|$, [2]

(ii) the conjugate of ab, giving your answer in the form $x + iy$. [2]

State clearly the relationship between $\text{Arg}(a)$, $\text{Arg}(b)$ and $\text{Arg}(ab)$. [1]

9 **(a)** The complex number $z = x + iy$ satisfies the equation $\frac{z}{z+2} = 2 - i$.

Find the value of x and the value of y. [4]

The complex number $\frac{3+2i}{2+ai}$ can be expressed in the form $x + iy$, where x and y are real. Find the value of a given that $x = y$. [5]

(nov2007)

3

3 Given the complex number $W = 2 - 3i$,

evaluate

(i) iW,

(ii) $W + iW$. [3]

Plot the points P, Q and R representing the complex numbers W, iW, $W + iW$ respectively on an Argand diagram. [2]

Hence name the quadrilateral OPRQ, where O is the origin. [1]

1. The complex number $p = 3 - 5i$ and it is given that $q = 4ip$.

(a) State the relationship between

(i) $|p|$ and $|q|$,

(ii) $\arg(p)$ and $\arg(q)$. [2]

(b) Given that $r = p + q$, find r in the form $a + bi$ where a and b are real numbers. [2]

(c) The points P, Q and R in an Argand diagram represent the complex numbers p, q and r, respectively.

(i) State the kind of quadrilateral that OPRQ is, where O is the origin. [1]

(ii) Find the area of OPRQ. [3]

1 A complex number z has modulus 8 and argument $\frac{3\pi}{4}$.

State the modulus and argument of z^2. [2]

Using these values show the number z^2 on an Argand diagram, and hence express z^2 in the form $a + bi$. [2]

1. The complex number $p = 3 - 5i$ and it is given that $q = 4ip$.

(a) State the relationship between

(i) $|p|$ and $|q|$,

(ii) $\arg(p)$ and $\arg(q)$. [2]

(b) Given that $r = p + q$, find r in the form $a + bi$ where a and b are real numbers. [2]

(c) The points P, Q and R in an Argand diagram represent the complex numbers p, q and r, respectively.

(i) State the kind of quadrilateral that OPRQ is, where O is the origin. [1]

(ii) Find the area of OPRQ. [3]

5 The complex numbers z and w are given by $z = -3 + 2i$ and $w = 5 + 4i$.

Find

(i) $|z|$, [1]

(ii) $\arg z$, [2]

(iii) $\frac{z}{w}$ in the form $a + ib$ where a and b are exact.

Hence represent $\frac{z}{w}$ in an Argand diagram. [3]

1. The complex number $z_1 = 1 - 2i$ and the complex number z_2 is such that $z_1 z_2 = -10i$.
   Find z_2 in the form $a + ib$ and sketch it on an Argand diagram. [5]

3 Express $z = \frac{1+i}{3+4i}$ in the form $a + bi$, where a and b are real. [3]

Hence or otherwise find $|z|$ in the form $c\sqrt{d}$ where d is a prime number. [2]

It is given that $z_1 = 2 - 4i$ and $z_2 = 6 - 2i$

(a) Find $z_1 - z_2$ and $z_1 z_2$ in the form $a + bi$. [3]

(b) If $w = \frac{1}{z_1}$, obtain the exact values of the modulus and argument of w. [4]

10 (a) The complex number u is such that $(-4 + 3i)u = 5 - 3i$.

Find

(i) the modulus of u,

(ii) the argument of u. [4]

(b) Given that the complex number w is $2i$.

Find in the form $a + ib$

(i) $\frac{u}{w}$,

(ii) uw. [4]

The complex number $w = \frac{4 + 3i}{3 - 2i}$.

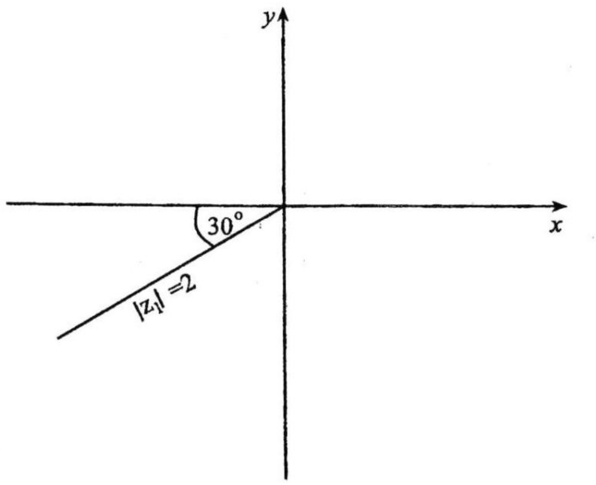
(a) Express w in the form $x + iy$ where x and y are real. [2]

(b) Find,

(i) modulus of w,

(ii) argument of w. [5]

7



A complex number z_1 has modulus 2 and is positioned as shown in the Argand diagram above.

(i) State the principal argument of z_1 and write z_1 in the form $a + ib$ where a and b are exact real numbers. [3]

(ii) Find exactly in the form $a + ib$, the complex number w, given that

$$w = \frac{\left(-8\sqrt{3}\right)i}{z_1}.$$ [2]

(iii) Show a sketch of w in an Argand diagram, labelling the modulus and argument values in your diagram. [3]

11 Given that $p = 5 + \text{i}$ and $q = -2+3\text{i}$,

(a) **(i)** show the complex numbers ip and $p+q$ on an argand diagram,

(ii) describe the geometrical transformation which maps ip onto p. {3]

(b) Find

(i) the modulus and argument of p,

(ii) pq,

(iii) $\frac{p}{q}$. {5]

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8 If $Z_1 = -1 + \text{i}$ and $Z_2 = -1 - \sqrt{3}\text{i}$,

find

(i) the modulus and argument of Z_2 [2]

(ii) **(a)** $Z_1 Z_2$,

(b) $\frac{Z_1}{Z_2}$. [4]

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13 Given that $a = 2 + i$ and $b = 1 + 3i$,

(i) show on a single argand diagram the complex numbers

1. ab,

2. $\frac{a}{b}$. [6]

(ii) find the modulus and argument of each case in **(i)**1 and **(i)** 2. [4]

10 The complex number z satisfies the equation

$$z + 2\bar{z} = \frac{13}{-2+3i}.$$

Find

(i) z in the form $x + iy$, [3]

(ii) the modulus and argument of $\frac{1}{z}$. [4]

7 The complex number $w = 3 - 4i$ and u is such that $\frac{w}{u} = \frac{2}{13} + \frac{3}{13}i$.

(a) Find

(i) u in the form $x + iy$,

(ii) 1. $|u|$,

2. $\arg u$. [7]

(b) Sketch u on an argand diagram showing clearly the $|u|$ and $\arg u$. [2]

Paper 1 Pure Mathematics 3 hours (120 marks)

1 Indices and proportionality
2 Polynomials
3 Identities, equations and inequalities
4 The modulus function
5 Graphs and coordinate geometry in two dimensions
6 Vectors (1)
7 Functions
8 Sequences and series
9 Series expansions
10 Plane trigonometry

11 Trigonometrical functions
12 Logarithmic and exponential functions
13 Differentiation
14 Integration
15 First order differential equations
16 Numerical methods
17 Complex Numbers (1)