1. Given that $f(x) = mx^3 + (m+2n)x^2 + (m+n)x + 2$ is exactly divisible by $(x+1)$, express n in terms of m. [2]

Show that, if in this case, the equation $f(x) = 0$ has only one real root, then $m^2 - 6m + 4 < 0$. [4]

(nov2003)

2. Given that $(x+2)$ is a factor of $x^4 + ax^2 + 3x + 6$, [2]
Find the value of a.

(june2004)

3. Given that $(x-1)$ and $(x+3)$ are factors of the polynomial $x^3 + ax^2 + bx - 9$, find the values of a and b. [3]

(nov2004)

4. Given that $(x+2)$ and $(2x-1)$ are factors of $f(x) = 2x^3 + ax^2 + bx + 6$, find the value of a and the value of b. [5]

Hence find the third factor. [2]

(june2005)

1 The polynomial $x^3 + px^2 + qx - 81$, where p and q are constants, has factors $(x+1)$ and $(x-3)$. Calculate the value of p and the value of q. [4]

13 Find the value of b for which $f(x) = 3x^4 + 19x^3 + 43x^2 + bx - 42$ is divisible by $(3x-2)$. [2]

Hence factorise $f(x)$ completely. [3]

Show that one of the factors of $f(x)$ is always positive for all x [3]

5. Find the value of k for which $(x+2)$ is a factor $f(x) = x^3 + 2x^2 - kx - 1$. Hence factorise $f(x)$ completely. [5]

3 Given that $(x-1)$ and $(x-b)$ are factors of the polynomial $f(x) = x^3 + 2ax^2 + bx - 1$, where $a > 0$ and $b > 0$, find the values of a and b. [5]

6. Find the value of k for which $(x+2)$ is a factor $f(x) = x^3 + 2x^2 - kx - 1$. Hence factorise $f(x)$ completely. [5]

6 Given that $(3x-2)$ is a factor of $6x^3 + 8x^2 + kx + 2$,

find the value of k. [2]

Hence find all the roots of the equation

$$6x^3 + 8x^2 + kx + 2 = 0,$$

giving your answers in exact form. [4]

7. Find the value of a for which $(x-2)$ is a factor of $3x^3 + ax^2 + x - 2$ [2]

Show that for this value of a the cubic equation $3x^3 + ax^2 + x - 2 = 0$ has only one real root. [3]

12 One root of the equation $x^4 + ax^3 + bx^2 + 16x - 12 = 0$ is 2 and the other root is -2.

Find

(a) the values of a and b, [4]

(b) the other two roots using the values of a and b found in **(a)** above. [4]

2 The cubic polynomial $f(x)$ is given by $f(x) = 2x^3 + 6x^2 + kx + 12$, where k is a constant. The function $f(x)$ leaves a remainder of 6 when divided by $x + 2$.

Find

(a) the value of k, [2]

(b) the solutions of the equation $f(x) = 9$, given that it has only one real root. [5]

8 The function $f(x)$ has equation $f(x) = 2x^3 + bx^2 + x - 2$.

Given that $2x - 1$ is a factor of $f(x)$,

(i) find the value of b, [2]

(ii) factorise $f(x)$ completely. [3]

(iii) Sketch the curve of $y = f(x)$, showing clearly the points of intersection of the curve with the axes without necessarily finding the coordinates of the turning points. [2]

Hence, or otherwise, find the values of x which satisfy the inequality

$$f(x) < 0.$$ [2]

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5 When a polynomial $f(x) = x^3 + ax^2 + bx + c$ is divided by $x^2 - 4$ the remainder is $2x + 11$. If $x + 1$ is a factor of $f(x)$, find the values of a, b and c. [5]

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5 Given that $(x + k)$ is a factor of $x^3 + 2x^2 - 3x - 6$, where $k > 0$,

Find

(i) the value of k, [2]

(ii) the exact roots of the equation $x^3 + 2x^2 - 3x - 6 = 0$. [3]

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5 The polynomials $P(x) = x^3 - x^2 + 4x$ and $Q(x) = x^3 + 6x - 10$ leave the same remainder when divided by $x - a$.

(a) Find the possible values of a. [3]

(b) Solve the inequality $P(x) > Q(x)$. [2]

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5 **(a)** Given that $f(x) = -x^3 + 2x^2 + 3x - 6$,

(i) factorise $f(x)$ completely,

(ii) sketch the curve $y = f(x)$ showing all the intersections with the axes. [6]

[You need not find the turning points]

(b) Hence or otherwise, write down the solution of the inequality $f(x) > 0$. [2]