1. Given that f(x) = $\frac{x+4}{x+1}$, describe the geometric transformations required to obtain the graph of f(x) from the graph of $y = \frac{1}{x}$. [4]

(nov2003)

2. Functions of f and g, each with domain IR are defined as follows:

$f : x \mapsto 3x + 2$ g: $x \mapsto x^2 + 1$.

(i) State the range of f and of g. [1]

(ii) Give a reason to show whether or not each function is one to one. [2]

(iii) Find the values of x for which $f\text{g}(x) = \text{g}f(x)$. [3]

(june2004)

3. Given the functions

$f: x \mapsto 4x - 10$ and

$g: x \mapsto \frac{8}{x}, x \neq 0$.

Express,

(a) in a similar fashion

(i) $f \circ g$ [1]

(ii) $g \circ f$ [1]

(iii) g^{15} [Hint $g^3 = g \circ g \circ g$] [2]

(b) in terms of one or both of f and g

(i) $x \mapsto \frac{1}{4}(x + 10)$ [1]

(ii) $x \mapsto 16x - 50$ [1]

(nov2004)

4. (a) Given that $\frac{4}{x} = 3 - \sqrt{5}$, show that $x + \frac{4}{x} = 6$. [3]

(b) Given that $f(x) = e^{2x-3}$ for all real values of x, find an expression for $f^{-1}(x)$, stating the domain for $f^{-1}(x)$, clearly. [4]

(june2005)

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5 **(a)** The functions f, g and h are defined by

$$f : x \mapsto \frac{x}{2x-3}, \quad x \in \mathbb{R}, x \neq \frac{3}{2};$$

$$g : x \mapsto \frac{1}{x+1}, \quad x \in \mathbb{R}, x \neq -1;$$

$$h : x \mapsto 3x+1, \quad x \in \mathbb{R}.$$

Find

(i) $hg(x)$, [1]

(ii) $f^{-1}(x)$. [2]

(b) The function f is defined by

$$f : x \to |5-x| \text{ for } -1 \leq x \leq 7.$$

(i) Sketch the graph of $y = f(x)$. [2]

(ii) State the range of f. [1]

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1. Show that $(x-3)$ is a factor of the function $f(x) = x^3 - 4x^2 - 3x + 18$. [2]
Factorize $f(x)$ completely. [2]
Hence sketch on separate diagrams, the graphs of

(i) $y = f(x)$, showing clearly the coordinates of the points at which the graph meets the axes. [1]

(ii) $y = f(x+3)$, showing clearly the coordinates of the points at which the graph meets the $x-$ axis. [1]

(iii) $y = f(x) - 4$, showing the coordinates of the point at which the graph meets the $y-$ axis. [1]
State, without solving, the roots of the equation $f(x) - 4 = 0$. [1]

2. Express $2x^2 - 6x + 3$ in the form $a(x+b)^2 + c$. [3]

(a) Hence, state

(i) the coordinates of the turning point of the graph of $y = 2x^2 - 6x + 3$, [2]

(ii) a sequence of three transformations by which the graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 6x + 3$. [3]

(b) Find the values of k for which the equation $2x^2 - 6x + 3 = kx + 1$ has distinct real roots. [4]

A function is defined by

$$f: x \mapsto x^2 + 4x + 1 \text{ for } x \geq -2.$$

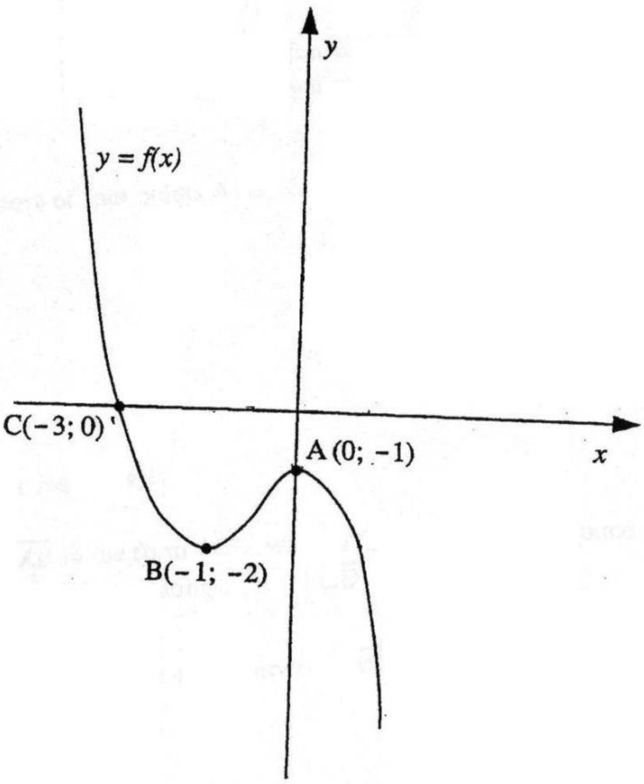
Find,

(i) the range of the function, [2]

(ii) an expression for $f^{-1}(x)$, stating its domain. [4]

(nov2007)

2 The diagram shows the graph of $y = f(x)$.



The points A, B and C have coordinates (0; −1); (−1; −2) and (−3; 0) respectively.
Sketch on separate axes, the graphs of

(i) $y = -f(x)$,

(ii) $y = f(x) + 2$.

Show clearly in each case the coordinates of the points corresponding to A, B and C. [4]

1. The function f is defined by $f: x \mapsto x^2 - 4x,\ x \in IR,\ |x| \leq 1$.

Show by means of a graphical argument or otherwise, that f is one-one, and find an expression for $f^{-1}(x)$. [5]

(june2009)

1. The function f is defined by $f: x \mapsto x^2 - 4x,\ x \in IR,\ |x| \leq 1$.

Show by means of a graphical argument or otherwise, that f is one-one, and find an expression for $f^{-1}(x)$. [5]

4 The function f is defined by

$$f : x \to \frac{1+x^2}{x}, x \in \mathbb{R}, x \neq k$$

(i) State the value of k. [1]

(ii) Find $f\left(\frac{1}{x}\right)$ in its simplest form. [2]

(iii) Find another element in the domain of f which has the same image as 2. [3]

1. The function f is defined as,

$$f : x \to \frac{2-x}{x+1}, \quad x \neq -1.$$

(a) (i) Express $f(x)$ in the form $a + \frac{b}{x+1}$, where a and b are constants. [2]

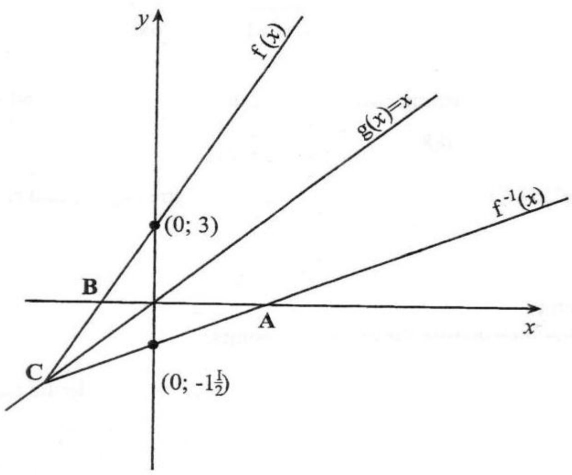
(ii) Hence, give a sequence of three transformations which take the graph of $y = \frac{1}{x}$ onto the graph of $y = f(x)$ [3]

(iii) State the range of f. [1]

(b) (i) Form the composite function $f\big(f(x)\big)$. [2]

(ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$. [1]

7 The diagram below shows the linear graphs of $f(x)$, $g(x) = x$ and $f^{-1}(x)$ which intersect at **C**. The graph $f^{-1}(x)$ intersects the x-axis at point **A** and $f(x)$ intersects x-axis at **B**.

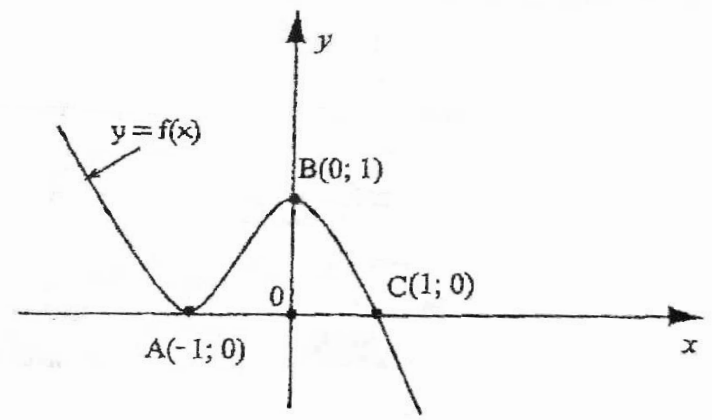


(a) State the name given to the function $g(x)$ in relation to the functions $f(x)$ and $f^{-1}(x)$. [1]

(b) Write down the coordinates of the points **A** and **B**. [3]

(c) Calculate the coordinates of the point **C**. [3]

7.



The diagram shows the graph of $y = f(x)$. Sketch, on separate diagrams showing the images of A, B and C the graphs of

(i) $y = f(2x)$
(ii) $y = f(-x) + 3$
(iii) $y = 2f(x + 1)$ [6]

---

3 **(a)** Describe the transformations needed to transform the graph of $y = \sin x$ onto the graph of $y = 6\sin(2x) - \pi$. [3]

(b) A curve has equation $y = \frac{x^2 - 4}{x + 1}$.

Find the equation of the normal to the curve at P(2; 0) in the form $ax + by + c = 0$, where a, b and c are integers. [4]

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8 The function $h(x) = 2x^2 - 6x + 11,\ x \in \mathbb{R}$.

Express

(a) $h(x)$ in the form $a(x + b)^2 + c$ where a, b and c are constants. [2]

(b) Write down the range of h. [1]

(c) Explain why h does not have an inverse. [1]

(d) Given that $h(x)$ is defined for $x \in$ A, where A$\subset\mathbb{R}$, find

(i) the largest element of A for which $h(x)$ has an inverse,

(ii) $h^{-1}(x)$ for this set of elements of A. [4]

14 **(a)** Functions f and h are defined as follows:

$f: x \rightarrow (x-2)(x+3) \quad x \in \mathbb{R} \; x \geq -\frac{1}{2}$

$h: x \rightarrow 4x^2 + 1 \quad x \in \mathbb{R}$

Find

(i) the exact values of x for which $fh(x) = 0$,

(ii) the inverse of $f(x)$ and state its domain. [6]

4 The functions f and g are defined for $x \in \mathbb{R}$ by

$f: x \rightarrow x^2 + 4x + 1$
$g: x \rightarrow ax + b.$

Given that $fg(2) = -2$ and $gf(0) = -3$, find the values of a and b. [5]

9 The function $f(x) = \sin\left(x - \frac{\pi}{4}\right)$ for $0 \leq x \leq 2\pi$.

(i) Sketch on separate diagrams, showing clearly the intercepts with the x-axis (if any), the graphs of

1. $y = f(x)$,

2. $y = f(2x)$,

3. $y = 2 + f\left(x + \frac{\pi}{4}\right)$.

[5]

(ii) Describe fully the geometric transformations which the graph of $y = f(x)$ undergoes to obtain the graphs in part **(i)** 2 and part **(i)** 3. [4]