1. Expand $(8+x)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3, simplifying the coefficients.

State the values of x for which the expansion is valid. [5]

(nov2003)

2. Find the coefficient of x^6 in the expansion of $(9+3x^2)^{\frac{1}{2}}$ in ascending powers x. [3]

(june2004)

3. (i) Expand $(1-x)^{-\frac{1}{2}}$ up to and including the term in x^3, simplifying your coefficients. [3]

(ii) Hence by putting $x = \frac{1}{50}$, estimate $\sqrt{2}$ correct to six decimal places. [4]

(nov2004)

4. Expand $(1+2x)^4$ in ascending powers of x. [2]

Hence find the coefficient of the term in x^3 in the expansion of $(2x+3)(1+2x)^4$. [2]

(june2005)

7 **Use the binomial expansion to show that when x is sufficiently small for x^3 and higher powers of x to be neglected,**

$$\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{x^2}{2}.$$ **[5]**

By putting $x = \frac{1}{8}$, show that $\sqrt{7} \approx \frac{339}{128}$. **[3]**

10 Write down the first five terms of the expansions of $ln(1+x)$ and $ln(1-x)$. [2]

Deduce that $ln\left(\frac{1+x}{1-x}\right) \approx 2\left(x+\frac{1}{3}x^3+\frac{1}{5}x^5\right)$ for values of x which are small enough for terms in x^6 and higher powers to be neglected. [2]

By using the substitution $x = \frac{1}{N}$,

show that $ln\left(\frac{1+\frac{1}{N}}{1-\frac{1}{N}}\right) \approx 2\left(\frac{1}{N}+\frac{1}{3N^3}+\frac{1}{5N^5}\right)$, where $\frac{1}{N}$ is small. [1]

Hence write down the expansion of $ln\left(\frac{1+\frac{1}{N}}{1-\frac{1}{N}}\right)^N$ in ascending powers of $\frac{1}{N}$. [2]

Deduce that, for large N, $\left(\frac{1+\frac{1}{N}}{1-\frac{1}{N}}\right)^N \approx e^2$. [2]

4 Obtain the first four terms of the expansion of $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, simplifying the coefficients. [3]

State the range of values of x for which the expansion is valid. [1]

By putting $x = 0.0002$ in your expansion, find $\frac{1}{\sqrt[3]{0.9994}}$ correct to 11 decimal places. [2]

11 **(i)** By first expanding $\cos(2x + x)$, prove that $\cos 3x = 4\cos^3 x - 3\cos x$. [3]

(ii) Use the series expansion of $\cos x$ given in the list of formulae to express $3\cos x$ and $\cos 3x$ in terms of powers of x, upto and including the term in x^6. Hence show that $\cos^3 x = 1 - \frac{3}{2}x^2 + \frac{7}{8}x^4 - \frac{61x^6}{240} + \ldots$ [4]

(iii) Calculate the relative error in using $1 - \frac{3}{2}x^2 + \frac{7}{8}x^4 - \frac{61x^6}{240}$ as an approximation for $\cos^3 x$ when $x = \frac{1}{3}\pi$, giving your answer to 1 significant figure. [3]

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1. Find the coefficient of x^6 in the expansion of $(9 + x^2)^{\frac{1}{2}}$ in ascending powers of x. [3]

(a) Show that $\left(1 - \frac{1}{n^2}\right)^{-1} = \left(\frac{n^2}{n^2 - 1}\right)$. [2]

(b) (i) Write down the first three terms of the series expansion of $\ln(1 - x)$ in ascending powers of x, up to and including the term in x^3. [1]

(ii) Hence write down the first three terms for $\ln\left(1 - \frac{1}{n^2}\right)$, in terms of n, and deduce the first three terms in the expansion of $\ln\left(1 - \frac{1}{n^2}\right)^{-1}$. [1]

(i) Show that $\ln\left(\frac{n^2}{n^2 - 1}\right) = 2\ln(n) - \ln(n - 1) - \ln(n - 1)$. [2]

(ii) Given that $\ln 10 = 2.302\ 585\ 1$, $\ln 3 = 1.098\ 612\ 3$, and $n = 10$, and using your results in (ii) and (iii), calculate $\ln 11$, to six decimal places. [4]

(nov2007)

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8 Find a quadratic expression $ax^2 + bx + c$ such that $1 - x^3 \equiv (1 - x)(ax^2 + bx + c)$. [2]

Hence obtain an expression for $ln(ax^2 + bx + c)$ as a difference of two logarithms and use it to find an expansion of $ln(ax^2 + bx + c)$ in ascending powers of x up to and including the term in x^5. [5]

7 For each of the following expressions, obtain a series of simplified terms, in ascending powers of x, up to and including the term in x^3 stating the set of values for which each expansion is valid.

(a) $(1-4x)^{-\frac{1}{4}}$. [4]

(b) $ln(5-3x)$. [4]

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1 Find the coefficient of x^{-3} in the expansion of $\left(x^2-\frac{2}{x}\right)^9$ [4]

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1. (a) Find the term independent of x in the expansion of

$$\left(x^2+\frac{3}{x}\right)^6$$ [4]

(b) Find the series expansion of $(4+x^2)^{\frac{1}{2}}$ up to and the term in x^6. [4]

Find the expansion of $\sqrt{(4-3x^2)}$ up to and including the term in x^4. [4]

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9 It is given that $f(x)=\frac{1}{(1+x)^2}+\sqrt{9+x}$.

(a) Expand $f(x)$ up to and including the term in x^2 simplifying coefficients. [7]

(b) State the values of x for which the expansion is valid. [1]

If the expansion of $(1+x)^{\frac{1}{5}}$ and of $\frac{1+px}{1+qx}$ in ascending powers of x are identical up to and including the term in x^2, calculate the value of p and the value of q. [7]

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6 **(i)** Express $\frac{4x+6}{(x+2)(x+1)(x+3)}$ in partial fractions. [3]

(ii) By using series expansion up to and including the term in x^2 show that $\frac{4x+6}{(x+2)(x+1)(x+3)}$ can be reduced to $1-\frac{7}{6}x+\frac{41}{36}x^2$. [4]

(b) Find the series expansion of $\dfrac{(4-x)^{\frac{1}{2}}}{2x^2-1}$ up to and including the term in x^2.

State the range of values of x for which the expansion is valid. [5]

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2 Find the coefficient of x^2 in the expansion of $\left(x - \dfrac{2}{5x}\right)^6$. [3]

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13 **(i)** Use the binomial expansion to simplify $(x+2)^7 - (x-2)^7$. [6]

(ii) Hence use your answer in **(i)** to find the exact value of $\left(\sqrt{5}+2\right)^7 - \left(\sqrt{5}-2\right)^7$. [2]

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2 **(i)** The expansion of $(1+ax)^n$ up to and including the term in x^2 is $1 - 6x + \dfrac{81}{4}x^2$.

Find the values of a and n. [5]

(ii) Hence state the values of x for which the expansion is valid. [1]